Name ______________________________ Date ____________________

Math 10250 Activity 4: Limits (Section 1.1)

GOAL: To obtain an intuitive understanding of the fundamental concept of limit and learn rules for computing it.

Q1: Using your intuition, how would you interpret the statement: The function $f(x) = \frac{x^2-2x-3}{x-3}$ has limit 4 as x goes to 3? *$f(x) \approx 4$ when x is close to 3 but $x \neq 3$* "$\approx$" *reads is approximately equal to*

A1: Natural domain of f: $x \neq 3$.

Since f is not defined at $x = 3$, let's look at how f behaves near $x = 3$. To do this, we make a table of values like this:

Step 1: We do numerical experimentation:

x	2.97	2.98	2.99	3	3.01	3.02	3.03
$f(x) = \frac{x^2-2x-3}{x-3}$	3.97	3.98	3.99	?	4.01	4.02	4.03

Pattern: $f(x)$ gets close to _4_ as x gets close to 3.

To make this more precise we need the help of algebra. So, let us factor the numerator of f:

Step 2: Simplify $f(x)$ to remove $x-3$:

$$f(x) = \frac{x^2-2x-3}{x-3} = \frac{(x-3)(x+1)}{x-3} \overset{x\neq 3}{=} x+1$$

Step 3: Send x to 3 in the simplified form:

$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3}(x+1) = 3+1 = 4, \text{ i.e.}$$

- *$f(x) \approx 4$ for all x near 3 (but $x \neq 3$), and*
- *can make $f(x)$ as close to x as we wish by taking x close enough to 3*

Sketch of $y = f(x)$:

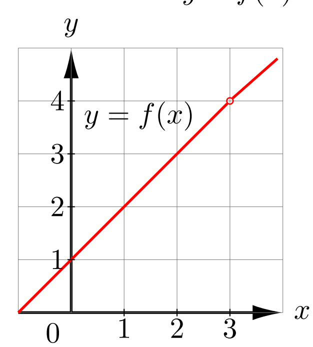


Figure 1

Now, we are confident to claim that the limit of $f(x)$ as x goes to 3 is 4.

We write this as: $\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = 4.$

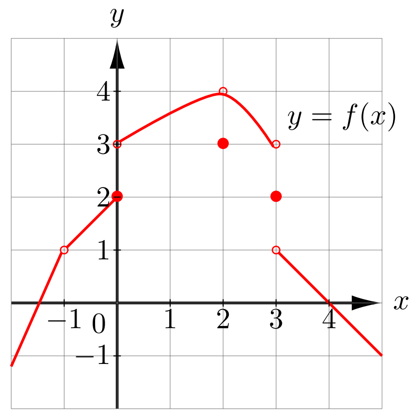
Q2: Give an **Informal Definition of Limit**.

A2:

$\lim_{x\to a} f(x) = L$, *if:*

- *$f(x) \approx L$ if x is close to a (but not equal to a)*
- *can make $f(x)$ as close to L as we wish by taking x close enough to a*

Exercise 1 The graph of a function f is shown in Figure 2. By visually inspecting the graph, find each of the following limits if it exists. If the limit does not exist, explain why.



(i) $\lim_{x\to 4} f(x) \stackrel{?}{=} 0$

(ii) $\lim_{x\to -1} f(x) \stackrel{?}{=} 1$

(iii) $\lim_{x\to 2} f(x) \stackrel{?}{=} 4$

(iv) $\lim_{x\to 0} f(x) \stackrel{?}{=}$ does not exit

(v) $\lim_{x\to 3} f(x) \stackrel{?}{=}$ does not exit

Exercise 2 Find $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$. Complete the following table of values to guess the limit and then use algebra to justify it (as in **A1**).

Step 1. We do numerical experimentation in the following table:

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$\dfrac{x^2-4}{x-2}$	3.9	3.99	3.999	**?**	4.001	4.01	4.001

Step 2. $\dfrac{x^2-4}{x-2} = \dfrac{(x-2)(x+2)}{x-2} = x+2$

Step 3. $\lim_{x\to 2} \dfrac{x^2-4}{x-2} = \lim_{x\to 2}(x+2) = 2+2 = 4$

Q3: What are the basic **Limit Laws**?

A3:

0. $\boxed{\lim_{x\to a} x^n = a^n}$

1. $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

2. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

3. $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

4. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

5. $\lim_{x\to a}[f(x)]^r = \left[\lim_{x\to a} f(x)\right]^r$

Exercise 3 Determine the following limits using the properties of limits (i.e., limit laws) and by simplifying the expression, if necessary.

(i) $\lim_{x\to 5} x^4 \stackrel{?}{=} 5^4$

(ii) $\lim_{x\to 2}(5x^3+4x^2) \stackrel{?}{=} 5\lim_{x\to 2} x^3 + 4\lim_{x\to 2} x^2 = 5\cdot 2^3 + 4\cdot 2^2$

(iii) $\lim_{x\to 2}\,(5x^3+4x^2)\cdot(x^2-9) \stackrel{?}{=} \left[\lim_{x\to 2}(5x^3+4x^2)\right]\cdot\left[\lim_{x\to 2}(x^2-9)\right] = \cdots$

(iv) $\lim_{x\to 2} \dfrac{x^2-9}{x-3} \stackrel{?}{=} \lim_{x\to 2} \dfrac{x^2-9}{x-3} = \dfrac{\lim_{x\to a} 2^2-9}{\lim_{x\to a} 2-3} = \dfrac{4-9}{-1} = \dfrac{-5}{-1} = 5$

(v) $\lim_{h\to 0} \dfrac{(h-2)^2-4}{h} \stackrel{?}{=} \lim_{h\to 0} \dfrac{h^2-4h+4-4}{h} = \lim_{h\to 0} \dfrac{h^2-4h}{h} = \lim_{h\to 0} \dfrac{h(h-4)}{h} = \lim_{h\to 0}(h-4) = -4$

Exercise 4 If $f(x)$ is the function of Exercise 1 and $g(x) = 3x+2$, then find the following limits:

(i) $\lim_{x\to 2}[f(x)\cdot g(x)] \stackrel{?}{=} \lim_{x\to 2} f(x) \cdot \lim_{x\to 2} g(x) = 4\cdot[3\cdot 2+2] = 32$ Ans. 32

(ii) $\lim_{x\to 2} \sqrt{f(x)} \stackrel{?}{=} \sqrt{\lim_{x\to 2} f(x)} = \sqrt{4} = 2$ Ans. 2